Paul's Notes on Phased Array Antennas

by

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Spatial Multiplexing

Spatial multiplexing refers to the transmission of multiple independent streams of data to different spatial positions or angles, using the same antenna aperture. This is easily achieved through digital beamforming.

Theoretically speaking, the maximum number of independently steerable beams is equal to the number of antenna elements. However in practice, we would not want too many beams, as it increases the peak-to-average ratio if the total emitted signal, which lowers the PA efficiency/increases chance of saturation.

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Dithering

Dithering is a technique whereby random noise is deliberated added to the transmitted signals of each of the antennas in a phased array, in order to spatially decorrelate the quantization noise.

When there is spatial correlation between the quantization errors, the error signal also adds coherently in the direction of the main beam, forming directive radiation.

Dithering decorrelates the quantization error, causing the energy to be spread out (ideally isotopically).

2.3.4 Miscalibration

Chapter 3

Array Signal Processing

3.1 Discrete Fourier Transform

The array factor of a phased array antenna can be efficiently computed using the Fast Fourier Transform (FFT), which is a method to compute the Discrete Fourier Transform (DFT). This is not only useful in evaluating the performance of the array, but also valuable in gaining intuition on the physics behind certain phenomena in phased array systems, such as the generation of grating lobes, and nonlinear spatial mixing.

3.2 Beamforming

Define the steering vector (also known as the array manifold vector) $\bar{v}_{\vec{k}}$ as

$$\bar{v}_{\vec{k}}[n] = e^{j\vec{k}\cdot\vec{r}_n} \tag{3.1}$$

Where $\vec{r}_n$ is the position of the n^{th} antenna. We can interpret the steering vector as the voltages measured at the positions of the antennas with an unit-amplitude plane wave incident *from* the direction of $\vec{k}$, or, in the direction of $-\vec{k}$.

In a transmitting array, the total transmitted signal to the direction of $\vec{k}$ is equal to $\bar{w}^{\dagger}\bar{v}_{\vec{k}}$, where $\bar{w}$ is a vector corresponding to the weights of the antennas.

When $\bar{v}_{\vec{k}}$ is parallel with $\bar{w}$, the response (transmitted or received power) is maximized. When the two vectors are orthogonal, the array response is minimized.

3.2.1 Conventional Beamformer (CBF)

The conventional beamformer for a beam towards the direction of $\vec{k}$ has weights equal to

$$\bar{w} = \bar{v}_{\vec{k}} \tag{3.2}$$

3.2.2 Minimum Variance Distortionless Response (MVDR) Beamformer

3.2.3 Linearly Constrained Minimum Variance (LCMV) Beamformer

Chapter 4

Multi-Input Multi-Output Arrays (MIMO)

4.1 Singular Value Decomposition (SVD)

Let $\vec{x}$ be an $N \times 1$ vector representing N streams of data.

$$\vec{x} = \begin{bmatrix} x_1(t) \\ \vdots \\ x_N(t) \end{bmatrix} \tag{4.1}$$

We transmit these data streams with an M-antenna phased array through $M \times N$ precoding matrix $\mathbf{B}$:

$$\vec{x}_t = \mathbf{B}\vec{x} \tag{4.2}$$

There are L receivers. The wireless channels are described by $L \times M$ channel matrix $\mathbf{H}$. The received signals $\vec{y}$ are given by

$$\vec{y} = (\mathbf{HB})\,\vec{x} + \vec{n}. \tag{4.3}$$

performing singular value decomposition on the (generally rectangular) matrix $\mathbf{HB}$ gives

$$\vec{y} = \mathbf{U\Sigma V}^{\dagger}\vec{x} + \vec{n}. \tag{4.4}$$

Recall that $\mathbf{\Sigma}$ is an $L \times N$ rectangular matrix with singular values of $\mathbf{HB}$. If we modify the precoding matrix as $\mathbf{B} \rightarrow \mathbf{BV}$, then the receive signal becomes

$$\vec{y} = \mathbf{U\Sigma}\vec{x} + \vec{n}. \tag{4.5}$$

If we process the received signals $\vec{y}$ by multiplying it with a matrix $\mathbf{U}^{\dagger}$, then we obtain a copy of the originally transmitted signals $\vec{x}$ propagated through a diagonal channel (and some noise):

$$\vec{y} = \mathbf{\Sigma}\vec{x} + \mathbf{U}^{\dagger}\vec{n}. \tag{4.6}$$

Note that each component of $\vec{x}$ may be scaled differently, but we can precompensate for it through further precoding with a diagonal power matrix $\mathbf{P}$.

So to summarize, on the transmitter side, we have total effective precoding matrix $\mathbf{BVP}$. On the receiver side, the signal is processed with the matrix $\mathbf{U}^\dagger$.